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# Estimation of Value-at-Risk on Romanian Stock Exchange Using Volatility Forecasting Models

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*This paper aims to analyse the market risk (estimated by Value-at-Risk) on the Romanian capital market using modern econometric tools to estimate volatility, such as EWMA, GARCH models. In this respect, I want to identify the most appropriate volatility forecasting model to estimate the Value-at-Risk (VaR) of a portofolio of representative indices (BET, BET-FI and RASDAQ-C). VaR depends on the volatility, time horizon and confidence interval for the continuous returns under analysis. Volatility tends to happen in clusters. The assumption that volatility remains constant at all times can be fatal. It is determined that the most recent data have asserted more influence on future volatility than past data. To emphasize this fact, recently, EWMA and GARCH models have become critical tools in financial applications. The outcome of this study is that GARCH provides more accurate analysis than EWMA.This approach is useful for traders and risk managers to be able to forecast the future volatility on a certain market.*

***Keywords:*** *Value-at-Risk, volatility forecasting, EWMA, GARCH models, autocorrelation*

## 1. Introduction

Value at Risk (VaR) is one of the widely used risk measures. VaR estimates the maximum loss of the returns or a portfolio at a given risk level over a specific period. VaR was first introduced in 1994 by J.P.Morgan and since then it has become an obligatory risk measure for thousands of financial institutions, such as investment funds, banks, corporations, and so on.

Classical VaR methods have several drawbacks. These methods include historical simulation, unconditional approaches and RiskMetrics. For instance, historical simulation method always assumes joint normality of the returns. On the other hand, the basic driving principle of the historical simulation method is its assumption that the VaR forecasts can be based on historical data. In the unconditional approach I use a standard deviation to estimate VaR and assume that the volatility constant over time. However, in reality these assumptions do not hold in most cases.

RiskMetrics measure the volatilty by using EWMA model that gives the heaviest weight on the last data. Exponentially weighted model give immediate reaction to the market crashes or huge changes. Therefore, with the market movement, it has already taken these changes rapidly into effect by this model. In

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this way EWMA responds the volatility changes and EWMA does assume that volatility is not constant through time.

The above models do not, however, incorporate the observed volatility clustering of returns, first noted by Mandelbrot (1963). The most popular model taking account of this phenomenon is the Autoregressive Conditional Heteroscedasticity (ARCH) process, introduced by Engle (1982) and extended by Bollerslev (1986).

Considering the above models, this study aims to estimate Value-at-Risk (VaR) of a portfolio of three representative indices on the Romanian capital market (BET, BET-FI and RASDAQ-C) using the most appropriate volatility forecasting model.

The data are daily (trading days) and cover the period from March 4, 2009 (date of the minimum reached on the capital market in Romania during the crisis) to November 30, 2013 (date of this study), for a total of 1218 daily observations.

The paper is structured as follows: The first part treats, from theoretical point of view, the concept and methodology of VaR and the volatility forecasting models. The second part presents the most relevant works in this field in Romania and abroad. The third part describes the data and methodology used. Also, results are interpreted. The last part summarizes the most important findings of the study.

## 2. Theoretical Framework

The VaR is a useful measure of risk. It was developed in the early 1990s by the corporation JPMorgan. According to Jorion (2001, p 19) "VaR summarizes the expected maximum loss over a target horizon with a given level of confidence interval."

In financial market, the typical time horizon is 1 day to 1 month. Time horizon is chosen based on the liquidity capabilitity of financial assets or expectations of the investments. Confidence level is also crucial to measure the VaR number. Typically in the financial markets, VaR number calculates between 95% to 99% of confidence level. Confidence level is choosen based on the objective such as Basel Committee requests 99% confidence level for banks regulatory capital.

In practice a variety of methods can be applied for calculation of VaR. These methods rely upon different assumptions. All VaR techniques can be divided into 2 broad categories:

a) *Historical approaches*, which rely on historical data and divide further on parametric and non-parametric models.

- Parametric models involve imposition of specific distributional assumptions on risk factors. *Log-normal approach* is the most widely used parametric model, which implies that market prices and rates are log-normally distributed. This kind of distribution is characterized only by 2 parameters: mean and standard deviation. Under the assumption of normality the VaR can be calculated as:

$$VaR = Z * \sigma * \sqrt{T}$$

where: Z - the quantile of normal distribution
T - holding period
σ – standard deviation of a risk factor

So, for the assessment of risk one needs only to know the volatility, which can be in turn estimated with the help of various techniques. The most popular are equally variance-covariance, weighted MA, EWMA and GARCH approaches. MA is simple a usual historical deviation, calculated over specific past period. EWMA on the other hand puts more weights on recent observations. This approach is justifiable when distant past influences the near future negligible (the situation of changing market conditions).

- Non-parametric approaches use historical data directly without any assumptions of risk factors' distributions. *Historical Simulation* is the easiest non-parametric model for practical implementation and assumes that risk factor volatility is a constant.

b) *Non-historical approaches* implies specific and explicitly given statistical model for distribution of the risk factors. Monte-Carlo simulation is a best-known representative of this class of models.

According to Allen (2004, p.54), Log-normal model involves estimation of risk factor distribution parameters using all available data. This approach assumes that risk factors are log-normally distributed.

Also, variance-covariance and weighted MA approaches use only the historical deviation and for this reason they are rarely applied in practice. Mostly EWMA and GARCH are used.

- *Exponentially Weighted Moving Average:*

In real life applications, some financial models assume the volatility is constant through time. This may be a mistake or can be misleading the results. According to Butler (1999, p. 190) "any financial assets that could currently have a lower volatility may have a much higher volatility in the future". In order to solve this problem, Butler (1999, p. 200) considers that risk mangers use EWMA model to give more weight on the latest data and less on the previous data.

Allen (2004) describes EWMA (exponential smoothing) as the improved method for predicting risk factor future volatility. Weights on more distant historical observations decline exponentially from initial weight to zero at the rate which is determine by decay factor (smoothing parameter).

This method was developed by J.P. Morgan (1996). The conditional volatility is estimated based on the following method:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2$$

where $\sigma_t^2$ is the forecast of conditional volatility, $\lambda = 0.94$ is the decay parameter ($\lambda$ is set at 0.94 for daily data as suggested in RiskMetrics), and $\varepsilon_{t-1}$ is the last period residual which follows the standard normal distribution.

$\varepsilon_t$ is a random variable (in this paper expressed in returns) with a zero mean and variance conditional on the past time series $\varepsilon_1, ..., \varepsilon_{t-1}$.

$\varepsilon_t = rt - \mu$

Where:

$rt$ - is continuous composed return of index at time *t;*

$\mu$ – is the mean of the returns

The VaR is calculated as follows:

$$VaR_t = Z_p\sigma_t$$

where $Z_p$ is the standard normal quantile\ for $p = 0.01; 0.05; 0.1; etc$

Note that EWMA estimation differs for various smoothing parameters. Under a weighting scheme with λ close to 1 recent information is more relevant and effective sample is shorter then under a weighting scheme with low λ. Optimal value of λ can be estimated using Maximum Likelyhood Method.

The RiskMetrics model is relatively easier to implement than other methods. However, the RiskMetrics model is subject to criticism because it ignores the asymmetric effect, the violation of the normality and risk in the tails of the distribution as often observed in the equity return data.

As a remedy, I can apply more complex and advanced models for determining the volatility to get a better proxy of the tail distribution. On the developed capital markets there are applied different models to estimate volatility.

Various advanced techniques for obtaining estimators of volatility have been continuously developed over the past period. They range from very simple models using the so-called random-walk assumptions to models regarding complex conditional heteroskedastic ARCH group (up to GARCH and derivatives thereof).

- *Heteroskedasticity models*

These models are divided into two categories: conditional models and unconditional models (or independent time variable). Although, there have been written a fairly extensive literature on the issue of independent volatility over time (homoskedasticity), practitioners have turned their attention to the second category approach of this issue, considering it more plausible, at least in terms of intuitive: volatility is not the same from one moment to another.

The most discussed univariate volatility models are autoregressive models with conditional heteroskedasticity (ARCH - *Autoregressive conditional heteroskedasticity*) proposed by Engle (1982) and the general GARCH (*Generalized Autoregressive conditional heteroskedasticity*) proposed by Bollerslev (1986). Many of these extensions have gained further importance as *Exponential* - GARCH (EGARCH) proposed by Nelson (1991), which empirically explains an asymmetric reaction of volatility to the impact of shocks in the market. Generally, each model has its own advantages and disadvantages, so, with a large number of models, all designed to serve to the same purpose, it is important to distinguish and correctly identify each model, with each features in order to establish the one who gives the best predictions. Jorion (2001, p. 170) states that the models for calculating VaR that use GARCH are more precise, principally in cases where there are volatility clusters.

In the following, I will make a brief presentation of these models.

**ARCH(1)**

The model was introduced in 1982 by the econometrician R. Engel in the journal Econometrica, and proposed a change in vision about how to estimate volatility. He said the standard deviation, by its way of calculating, gives equal weight (1 / n) to any historical observations considered in the determination of volatility.

Engel's model solves this inconvenience, giving more weight to the most recent observations and reducing weights of more distant observations. Thus, the variance (dispersion) from whose square root is resulting volatility, is expressed as:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2$$

where:

$\sigma_t^2$ - variance of the dependent variable in the current period;

$\omega$ - constant dispersion equation;

$\alpha$ - coefficient "ARCH";

$\varepsilon_{t-1}$ – residuals from the previous period;

**GARCH(1,1)**

It was proposed by T. Bollereslev (Engel's student) in 1986 in the Journal of Econometrics, and is part of a larger class of models GARCH (q, p). But it enjoys a great popularity among practitioners because of its relative simplicity. This model are similar to Engel's model. Variance formula is:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

where:

$\sigma_t^2$ - variance of the dependent variable in the current period;

$\omega$ - constant dispersion equation;

$\alpha$ - coefficient "ARCH";

$\varepsilon_{t-1}$ – residuals from the previous period;

$\sigma_{t-1}^2$ - variance of the dependent variable in the previous period;

$\beta$ - coefficient “GARCH”.

The model suggests that the variance forecast is based, in this case, on the most recent observation of assets return and on the last calculated value of the variance. The general model GARCH (q, p) calculates the expected variance on the latest q observations and the latest p estimated variances.

In the GARCH (1,1) model, described above, the first number shows that the residual terms of the previous period acts on dispersion and the second number shows that the dispersion of the previous period has influence on current dispersion. In fact, for very large series, GARCH (1.1) can be generalized to GARCH (p, q).

Because this application refers to volatility analysis of a selected portfolio, I will focus only on the dispersion equation. The model can be used successfully in volatile situations. GARCH model includes in its equation both terms and the phenomenon of heteroskedasticity. It is also useful if the series are not normally distributed, but rather they have "fat tails". No less important is that confidence intervals may vary over time and therefore more accurate intervals can be obtained by modeling of the dispersion of residual returns.

Different heteroskedastic volatility models (ARCH, GARCH, EGARCH, etc.) is based on historical prices. One advantage of these models from the implied volatility is given by the relatively recent research in finance, which shows a better estimation of the heteroskedasticity models from the initially more preferred implied volatility.

In this paper I use two univariate models: ARCH and GARCH in estimating VaR. VaR calculation consists of two steps:

- I forecast volatility using the models mentioned above;
- Calculate VaR based on the conditional volatility prediction:

$$VaR_t = Z_p \sigma_t$$

Where:

- $\sigma_t$ is the volatility estimated from heteroskedastic volatility models;
- $Z_p$ is p% quantile from the normal distribution.

After using different techniques in VaR estimation I need to check their predictive accuracy using various statistical tests. There are many VaR methodologies, and it is necessary to find the best model for risk forecasting. For the purposes of this paper, I explain and use "Violation ratio" of Danielsson (2011, p.145) for evaluating the quality of VaR forecasts.

If the actual loss exceeds the VaR forecast, then the VaR is considered to have been violated. The violation ratio is the sum of actual exceedences divided by the expected number of exceedences given the forecasted period. The rate is calculated as:

$$VR = \frac{Observed\ number\ of\ violations}{Expected\ number\ of\ violations} = \frac{E}{p\ x\ N}$$

Where:

- *E* is the observed number of actual exceedences
- *p* is the VaR probability level, in this case p=0.05 or 0.01
- *N* is the number of observations used to forecast VaR values.

## 3. Literature review

There are numerous research papers dedicated to analysis, development and practical application of the VaR methodology.

The VaR result could vary on the method chosen and the assumption of the correlation. Although VaR and other methods are accepted as effective risk management tools, they are not sufficient enough to monitor and control risk at all. The hope is to have only one powerful risk mesurment program that can solve the problems of investors and institutions, and able to measure risk effectively and systematically.

Jorion (2001) has mentioned the intricate parts of VaR calculations in his work. During the time when portfolio position is assumed to be constant that in reality does not apply to practical life. The disadvantage of VaR is it cannot determine where to invest. VaR simply illustrates the various speed of risk that are embbeded from the derivative instruments.

The second and third Basel Accord (International Convergence of Capital Measurement and Capital Standards, 2006 and Revisions to the Basel II Market Risk Framework, 2009) have laid down market risk capital requirements for trading books of banks. The market risk capital calculations can be done using either the standardized measurement method or the Internal Models approach. The internal models approach allows banks to calculate a market risk charge based on the output of their internal Value-at-risk (VaR) models.

Manganelli and Engle (2001) review the assumptions behind the various methods and discuss the theoretical flaws of each. The historical simulation (HS) approach has emerged as the most popular method for Value-at-risk calculation in the industry.

Hendricks (1996) compared twelve different VaR methods, namely equally weighted moving average (EQMA), exponentially weighted moving average (EWMA), and historical simulation (HS). For the 99% VaR it was observed that the HS approach provided better coverage than the other two VaR methods.

Hull and White (1998) improve the HS method by altering it to incorporate volatility updating. They adjust the returns using a conditional volatility model like GARCH or EWMA. According to these tests, the GARCH (1,1) model is suitable to estimate the conditional volatility, and is thus used to calculate the VaR.

In this paper I continue the scientific activity, aiming to identify the most appropriate volatility forecasting model to estimate the Value-at-Risk (VaR) of a portofolio of representative indices of Bucharest Stock Exchange. Given the emerging nature of the capital market in Romania, for representativity it was selected the period from the minimum reached during the Romanian capital market as a result of the recent financial crisis till the time of the present analysis.

The originality of our contribution to the current state of research in this field is generated by the following:

- I selected a portfolio of indices, so that it is included characteristics for the entire capital market in Romania (inclusion in the study of BET, BET-FI and RASDAQ-C indices);
- study was not just about applying a single methodology, being tested several models in order to select the most appropriate;
- study refers to recent years (though, being considered a representative number of observations) which determines the actuality of conclusions.

## 4. Data series and methodology

For portfolio construction, there were used data since March,04 2009 (date of the minimum reached on the capital market in Romania during the crisis) – to November, 30 2013 (date of this study), comprising a total of 1218 daily observations. I used in our analysis BET, BET-FI and RASDAQ-C indices.

The portfolio was selected with the following weights: 40% BET, BET-FI 30%, 30% RASDAQ-C. Criteria considered in determining these weights are based on the following assumptions: risk diversification by selecting indices whose composition covers a wide range of capital market in Romania, the weight of the average trading volume for the companies included in the indices.

In this paper, I use an out-of-sample VaR estimates to identify the most appropriate risk forecasting model. Out-of-sample VaR estimates are obtained based on the previous years'observations (values since March, 04 2009 to December, 31 2012) and are compared with the data from the last year (January, 02 2013 – November, 30 2013).

Based on primary data, there were calculated daily returns of the portfolio for the selected indices. Return was calculated using the following formula:

$$rt = \ln \frac{p_t}{p_{t-1}} = \ln p_t - \ln p_{t-1}$$

Where:

$rt$ is continuous composed return of index at time *t*, *pt* is the index value at time *t*.

The reason I've decided to use logarithmic returns in our study was highlighted by Strong (1992, p. 533) thus: "there are both theoretical and empirical reasons for preferring logarithmic returns. Theoretically, logarithmic returns are analytically more tractable when linking together sub-period returns to form returns over long intervals. Empirically, logarithmic returns are more likely to be normally distributed and so conform to the assumptions of the standard statistical techniques."

For this study, in the first phase I proceed to analyze the descriptive statistics of daily returns of selected indices and portfolio, then I apply various tests of normality and stationarity to highlight the characteristics of daily returns series. The next step will be to test the presence of ARCH signature in the indices portfolio. If I notice the signature ARCH, I will proceed to analyze the volatility through GARCH methodology. Finally, I will estimate the Value-at-Risk of the selected portfolio by all methods described in this study in order to select the most appropriate model.

For this analysis, I use as technical support the application Eviews7.

Next, I present a primary statistical data. In the following table I consider daily returns of BET, BET-FI and RASDAQ-C as well as portfolio selected.

***Table 1.*** *Descriptive Statistics*

| | DAILY_RETURN_BET | DAILY_RETURN_BET_FI | DAILY_RETURN_RASDAQ_C | DAILY_RETURN_PORTFOLIO |
|---|---|---|---|---|
| Mean | 0.001026 | 0.001133 | -0.000301 | 0.000989 |
| Median | 0.000845 | 0.000712 | 0.000187 | 0.000903 |
| Maximum | 0.105645 | 0.138255 | 0.048494 | 0.115302 |
| Minimum | -0.116117 | -0.149741 | -0.198265 | -0.132069 |
| Std. Dev. | 0.017065 | 0.025048 | 0.009654 | 0.021088 |
| Skewness | -0.169040 | 0.101382 | -8.846412 | -0.018240 |
| Kurtosis | 9.507520 | 8.073974 | 185.6334 | 8.188827 |
| | | | | |
| Jarque-Bera | 1712.639 | 1040.049 | 1357942. | 1085.985 |

| | | | | |
|---|---|---|---|---|
| Probability | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | | | | |
| Sum | 0.993280 | 1.096817 | -0.291609 | 0.957507 |
| Sum Sq. Dev. | 0.281601 | 0.606709 | 0.090122 | 0.430035 |
| | | | | |
| Observations | 968 | 968 | 968 | 968 |

Source: author calculations

The table also indicates that all 3 indices and the selected portfolio not follow a normal distribution. This fact is highlighted by the Skewness and Kurtosis indicator values.

Skewness normal distribution is zero. A positive Skewness series shows that the distribution is right asymmetry. For a negative Skewness, situation is reversed.

For normal distribution kurtosis (who shows "fat tails" or how much the maximum and minimum values deviate from their average) is 3.For K less than 3, distribution is flatter than normal (platykurtic) and for k greater than 3 distribution is higher (leptokurtic).

For the selected portfolio, skewness is –0.018 which shows an asymmetry to the left of distribution returns, sign that on certain days there were very high quotes. Kurtosis is 8.18 which indicates that the distribution is higher than normal. Jarque-Bera test value is 1085 and the attached test probability is 0%. Test values are quite far from the corresponding normal distribution, reason due to which I say that the series is not normally distributed.

This conclusion is strengthened by the following graphs: Histogram Graph and QQ-Plot Graph:

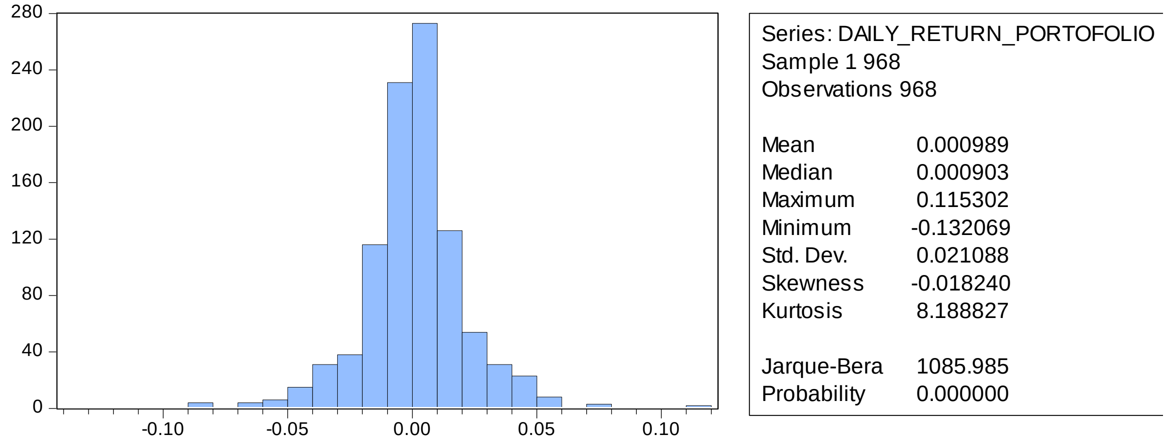


***Figure 1**. Histogram Graph*

Source: author calculations

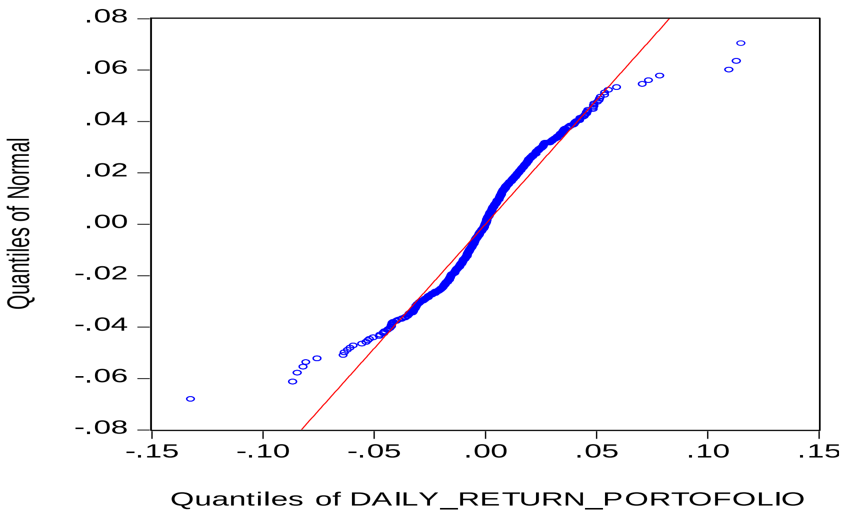


***Figure 2**. QQ Plot*

Source: author calculations

QQ-plot is a method used to compare two distributions, specifically, is the graph of the empirical distribution against a theoretical distribution (in this case, the normal distribution). If empirical distribution would be normal, should result QQ chart is first bisectrix, in this case is different from the normal distribution.

A more detailed inspection of the evolution of daily returns is performed using the following graph:

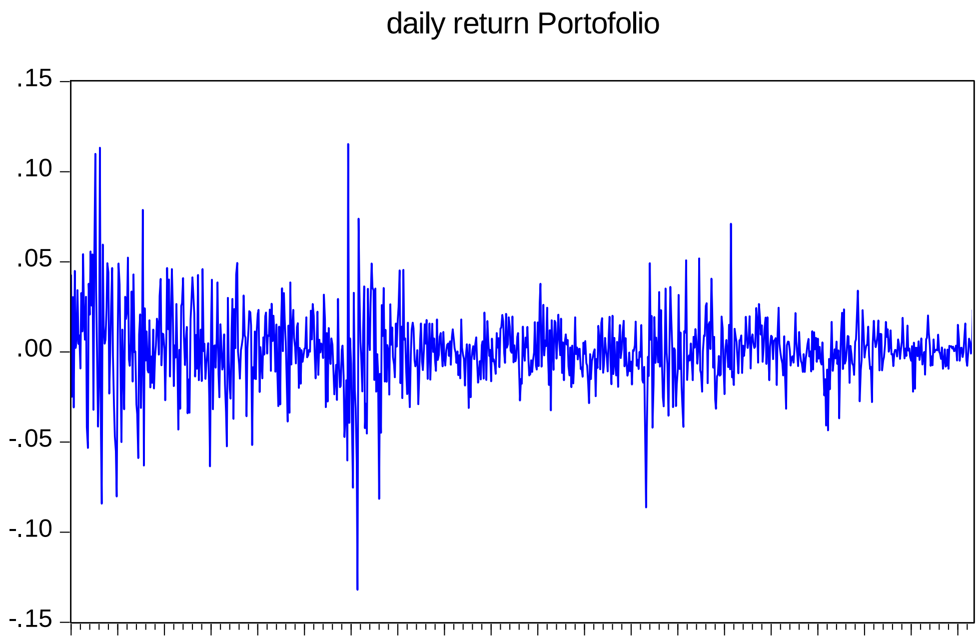


***Figure 3**. Returns Evaluation*
Source: author calculations

I see the chart above that there are pronounced extremities, another indication that the series is not normally distributed and an indication of possible "ARCH" signatures.

According to the ADF and Phillips-Perron tests, daily returns series are stationary for every level of relevance. Stationarity is defined as a quality of a process in which the statistical parameters (mean and standard deviation) of the process do not change with time. Otherwise, Shocks have transitory effects.

***Table 2.** ADF Test*

Null Hypothesis: DAILY_RETURN_PORTOFOLIO has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=21)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -29.19315 | 0.0000 |
| Test critical values: | 1% level | -3.436892 | |
| | 5% level | -2.864317 | |
| | 10% level | -2.568301 | |

Source: author calculations

***Table 3.** Phillips-Perron Test*

Null Hypothesis: DAILY_RETURN_PORTOFOLIO has a unit root
Exogenous: Constant
Bandwidth: 4 (Newey-West automatic) using Bartlett kernel

| | | Adj. t-Stat | Prob.* |
|---|---|---|---|
| Phillips-Perron test statistic | | -29.17246 | 0.0000 |
| Test critical values: | 1% level | -3.436892 | |
| | 5% level | -2.864317 | |

| | |
|---|---|
| 10% level | -2.568301 |

Source: author calculations

The above analysis is very useful in describing the series and economic phenomena. However, for certainty analysis, I test this ARCH signature with radical correlogram of daily returns. Number of lags used is 15. The column labeled AC remark serial correlation coefficients, while the last column I have the probability to accept the hypothesis "there is no ARCH effects" (which is actually null hypothesis). If I notice the signature ARCH, I will proceed to analyze the volatility through GARCH methodology.

***Table 4.*** *Correlogram of radical returns*

Sample: 1 968
Included observations: 966

| Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|
| *****\| \| | *****\| \| | 1 | -0.654 | -0.654 | 414.43 | 0.000 |
| \|* \| | ***\| \| | 2 | 0.161 | -0.466 | 439.55 | 0.000 |
| \| \| | **\| \| | 3 | -0.002 | -0.340 | 439.55 | 0.000 |
| \| \| | ***\| \| | 4 | -0.045 | -0.359 | 441.55 | 0.000 |
| \| \| | **\| \| | 5 | 0.072 | -0.294 | 446.58 | 0.000 |
| \| \| | *\| \| | 6 | -0.007 | -0.148 | 446.62 | 0.000 |
| *\| \| | *\| \| | 7 | -0.066 | -0.159 | 450.92 | 0.000 |
| \| \| | *\| \| | 8 | 0.052 | -0.167 | 453.55 | 0.000 |
| \| \| | *\| \| | 9 | 0.002 | -0.111 | 453.55 | 0.000 |
| \| \| | \| \| | 10 | -0.011 | -0.057 | 453.68 | 0.000 |
| \| \| | *\| \| | 11 | -0.010 | -0.067 | 453.78 | 0.000 |
| \| \| | \| \| | 12 | 0.014 | -0.051 | 453.96 | 0.000 |
| \| \| | \| \| | 13 | 0.004 | 0.004 | 453.98 | 0.000 |
| \| \| | \| \| | 14 | -0.025 | -0.029 | 454.59 | 0.000 |
| \| \| | \| \| | 15 | 0.035 | -0.010 | 455.78 | 0.000 |

Source: author calculations

Note that the null hypothesis probability value is 0, indicating that I can reject the null hypothesis and providing information there are ARCH effects.

The next step is finding the equation that best describes the portfolio volatility. In this respect, I estimate the equation of volatility with ARCH (1) and GARCH (1,1).

For volatility calculated by GARCH models, there was used Generalised Error Distribution (GED), given that the distribution is not normal series. The results are presented below.

***Table 5.*** *ARCH equation*

Dependent Variable: DAILY_RETURN_PORTOFOLIO
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Sample: 1 968
Included observations: 968
Convergence achieved after 7 iterations
Presample variance: backcast (parameter = 0.7)
ARCH = C(1) + C(2)*RESID(-1)^2

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| | | Variance Equation | | |
| C | 0.000236 | 2.13E-05 | 11.08085 | 0.0000 |
| RESID(-1)^2 | 0.531057 | 0.101489 | 5.232653 | 0.0000 |
| GED PARAMETER | 1.090728 | 0.062854 | 17.35325 | 0.0000 |
| R-squared | -0.002202 | Mean dependent var | | 0.000989 |

| | | | |
|---|---|---|---|
| Adjusted R-squared | -0.001167 | S.D. dependent var | 0.021088 |
| S.E. of regression | 0.021100 | Akaike info criterion | -5.177622 |
| Sum squared resid | 0.430982 | Schwarz criterion | -5.162512 |
| Log likelihood | 2508.969 | Hannan-Quinn criter. | -5.171870 |
| Durbin-Watson stat | 1.863669 | | |

Source: author calculations

To conclude if the above model is appropriate, I apply the Correlogram of Standardized Residuals.

***Table 6**. Correlogram of Standardized Residuals*

Sample: 1 968
Included observations: 968

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>| |</td><td>| |</td><td>1</td><td>-0.033</td><td>-0.033</td><td>1.0647</td><td>0.302</td></tr>
<tr><td>|* |</td><td>|* |</td><td>2</td><td>0.134</td><td>0.133</td><td>18.432</td><td>0.000</td></tr>
<tr><td>| |</td><td>|* |</td><td>3</td><td>0.066</td><td>0.075</td><td>22.610</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>4</td><td>0.039</td><td>0.027</td><td>24.094</td><td>0.000</td></tr>
<tr><td>|* |</td><td>|* |</td><td>5</td><td>0.139</td><td>0.126</td><td>43.049</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>6</td><td>0.046</td><td>0.045</td><td>45.128</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>7</td><td>0.072</td><td>0.040</td><td>50.213</td><td>0.000</td></tr>
<tr><td>|* |</td><td>|* |</td><td>8</td><td>0.138</td><td>0.120</td><td>68.779</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>9</td><td>0.006</td><td>-0.008</td><td>68.819</td><td>0.000</td></tr>
<tr><td>|* |</td><td>|* |</td><td>10</td><td>0.149</td><td>0.099</td><td>90.444</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>11</td><td>0.040</td><td>0.028</td><td>92.009</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>12</td><td>0.069</td><td>0.024</td><td>96.630</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>13</td><td>0.065</td><td>0.019</td><td>100.75</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>14</td><td>0.071</td><td>0.049</td><td>105.74</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>15</td><td>0.060</td><td>0.012</td><td>109.26</td><td>0.000</td></tr>
</table>

Source: author calculations

It is noted that all partial and total correlation coefficients exceed the limits, which indicates that there is correlation between residuals. Also, from the ARCH volatility chart, I see that volatility is not constant.

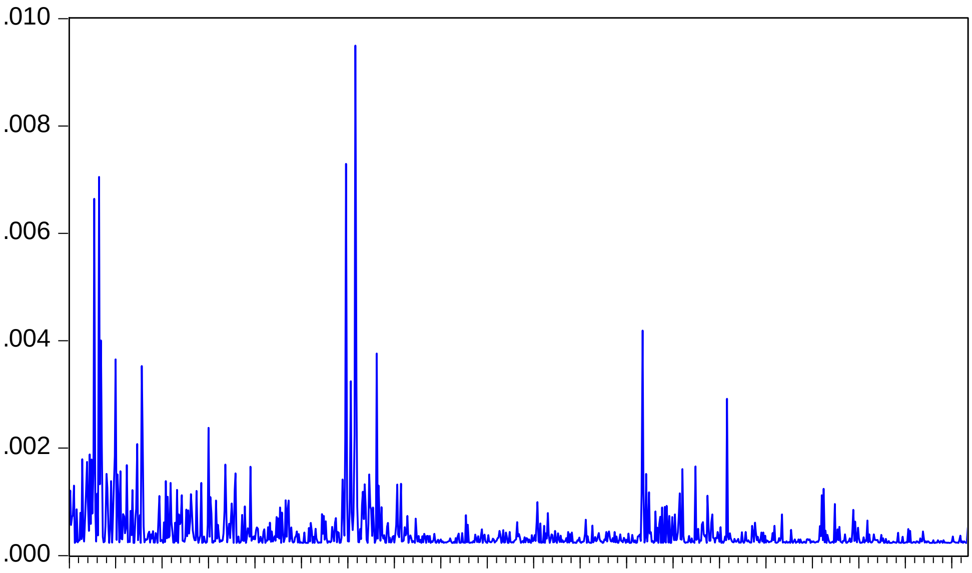


***Figure 4**. ARCH Graph*
Source: author calculations

For GARCH (1,1) I have the following equation:

***Table 7.*** *GARCH equation*

Dependent Variable: DAILY_RETURN_PORTOFOLIO
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Sample: 1 968
Included observations: 968
Convergence achieved after 13 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(1) + C(2)*RESID(-1)^2 + C(3)*GARCH(-1)

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| Variance Equation | | | | |
| C | 2.91E-06 | 1.48E-06 | 1.971756 | 0.0486 |
| RESID(-1)^2 | 0.099492 | 0.017018 | 5.846384 | 0.0000 |
| GARCH(-1) | 0.895441 | 0.016147 | 55.45594 | 0.0000 |
| GED PARAMETER | 1.362369 | 0.079337 | 17.17201 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | -0.002202 | Mean dependent var | 0.000989 |
| Adjusted R-squared | -0.001167 | S.D. dependent var | 0.021088 |
| S.E. of regression | 0.021100 | Akaike info criterion | -5.318961 |
| Sum squared resid | 0.430982 | Schwarz criterion | -5.298816 |
| Log likelihood | 2578.377 | Hannan-Quinn criter. | -5.311293 |
| Durbin-Watson stat | 1.863669 | | |

Source: author calculations

Following the results, I can highlight the following aspects:

- Coefficient of volatility C(1) is positive, indicating that when volatility increases, portfolio returns tend to increase;
- Coefficient C(2) that estimates ARCH effects in the data series analyzed, recorded a statistically significant amount. In other words, on the Romanian capital market, the periods characterized of high volatility continues throughout with high volatility, and vice versa.
- Coefficient C(3) which measures the asymmetry of the data series recorded a positive value, which suggests that negative shocks (bad news) generated less volatility than positive shocks (good news) on the Romanian capital market.

To validate this equation I apply the Correlogram of Standardized Residuals.

***Table 8.*** *Correlogram of Standardized Residuals*

Sample: 1 968
Included observations: 968

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>|* |</td><td>|* |</td><td>1</td><td>0.091</td><td>0.091</td><td>7.9941</td><td>0.005</td></tr>
<tr><td>| |</td><td>| |</td><td>2</td><td>0.029</td><td>0.020</td><td>8.7861</td><td>0.012</td></tr>
<tr><td>| |</td><td>| |</td><td>3</td><td>-0.006</td><td>-0.011</td><td>8.8230</td><td>0.032</td></tr>
<tr><td>| |</td><td>| |</td><td>4</td><td>0.023</td><td>0.024</td><td>9.3433</td><td>0.053</td></tr>
<tr><td>| |</td><td>| |</td><td>5</td><td>-0.007</td><td>-0.011</td><td>9.3967</td><td>0.094</td></tr>
<tr><td>*| |</td><td>*| |</td><td>6</td><td>-0.070</td><td>-0.070</td><td>14.128</td><td>0.058</td></tr>
<tr><td>| |</td><td>| |</td><td>7</td><td>-0.019</td><td>-0.006</td><td>14.472</td><td>0.063</td></tr>
<tr><td>| |</td><td>| |</td><td>8</td><td>0.017</td><td>0.022</td><td>14.761</td><td>0.064</td></tr>
<tr><td>| |</td><td>| |</td><td>9</td><td>-0.051</td><td>-0.056</td><td>17.343</td><td>0.054</td></tr>
<tr><td>| |</td><td>| |</td><td>10</td><td>0.028</td><td>0.040</td><td>18.117</td><td>0.053</td></tr>
<tr><td>| |</td><td>| |</td><td>11</td><td>0.028</td><td>0.025</td><td>18.863</td><td>0.064</td></tr>
<tr><td>| |</td><td>| |</td><td>12</td><td>-0.044</td><td>-0.060</td><td>20.801</td><td>0.053</td></tr>
<tr><td>| |</td><td>| |</td><td>13</td><td>-0.034</td><td>-0.025</td><td>21.953</td><td>0.056</td></tr>
</table>

| | | | | | | |
|---|---|---|---|---|---|---|
| \| \| | \| \| | 14 | 0.023 | 0.034 | 22.496 | 0.069 |
| \| \| | \| \| | 15 | -0.018 | -0.033 | 22.833 | 0.088 |

It is noted that partial and total correlation coefficients exceed the limits only for lag 1-3 and I can conclude that this model is quite suitable. The GARCH chart is the following:

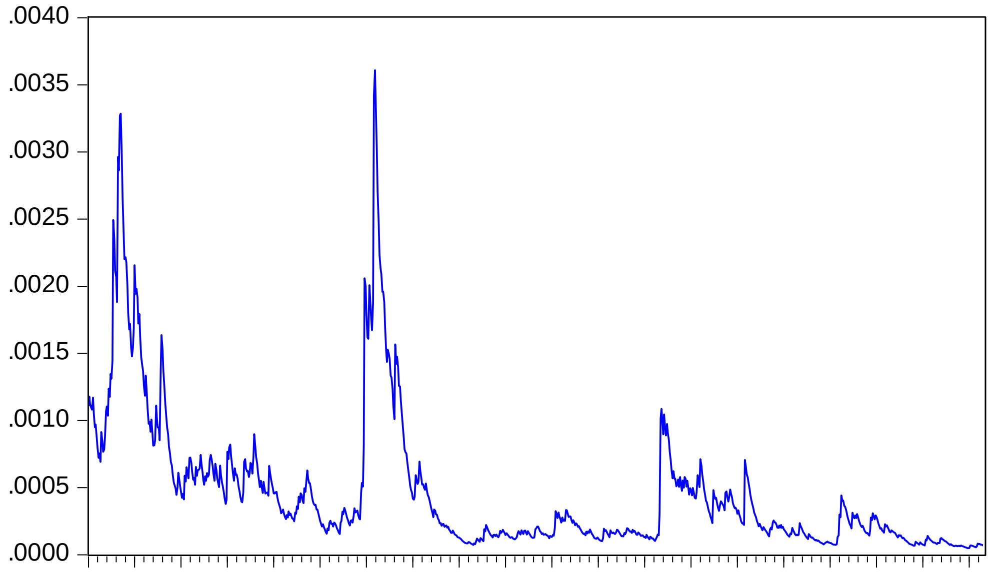


***Figure 5***. *GARCH Graph*
Source: author calculations

In the following, I'll estimate the VaR by the three models: EWMA, ARCH and GARCH.

- *Exponentially Weighted Moving Average:*

The VaR is calculated as follows:

$$VaR_t = Z_p \sigma_t$$

where $Z_p$ is the standard normal quantile for $p = 0.01; 0.05$;
The conditional volatility is estimated based on the following method (suggested by RiskMetrics) :

$$\sigma_t^2 = 0.94\sigma_{t-1}^2 + (1-0.94)\varepsilon_{t-1}^2$$

where $\sigma_t^2$ - variance of the dependent variable in the current period;
$\varepsilon_{t-1}$ - residuals from the previous period;

- *ARCH:*

The VaR is calculated as follows:

$$VaR_t = Z_p \sigma_t$$

where $Z_p$ is the standard normal quantile for $p = 0.01; 0.05$;
The conditional volatility is estimated based on the ARCH model:

$\sigma_t^2$ =0.000236 + 0.531056 $\varepsilon_{t-1}^2$

where:

$\sigma_t^2$ - variance of the dependent variable in the current period;

$\varepsilon_{t-1}$ – residuals from the previous period;

- *GARCH:*

The VaR is calculated as follows:

$$VaR_t = Z_p \sigma_t$$

where $Z_p$ is the standard normal quantile\ for $p = 0.01; 0.05$;

$\sigma_t^2$ =0.00000291 + 0.99492 $\varepsilon_{t-1}^2$ + 0.895441 $\sigma_{t-1}^2$

where:

$\sigma_t^2$ - variance of the dependent variable in the current period;

$\varepsilon_{t-1}$ – residuals from the previous period;

$\sigma_{t-1}^2$ - variance of the dependent variable in the previous period;

To find the best model for risk forecasting, I'll use the violation ratio of Danielsson (2011, p.145). For this reason I'll use an out-of-sample VaR estimates to identify the most appropriate risk forecasting model. This out-of-sample includes data from the last year (January, 02 2013 – November, 30 2013). If the actual loss exceeds the VaR forecast, then the VaR is considered to have been violated. The violation ratio is the sum of actual exceedences divided by the expected number of exceedences given the forecasted period. The confidence level is consider 95% and 99% and VaR is estimated daily.

$$VR = \frac{Observed\ number\ of\ violations}{Expected\ number\ of\ violations} = \frac{E}{p\ x\ N}$$

where - *E* is the observed number of actual exceedences

- *p* is the VaR probability level, in this case p=0.05 or 0.01

- *N* is the number of observations used to forecast VaR values, in this case 250 observations for year 2013.

Applying this methodology, I've obtained the following situation:

***Table 9.*** *Violation Ratio*

| | EWMA | | ARCH | | GARCH | |
|---|---|---|---|---|---|---|
| | **95%** | **99%** | **95%** | **99%** | **95%** | **99%** |
| **Violation Ratio** | 0.72 | 0.8 | 0 | 0 | 0 | 0 |

Source: author calculations

Graphically, the situation is as follows:

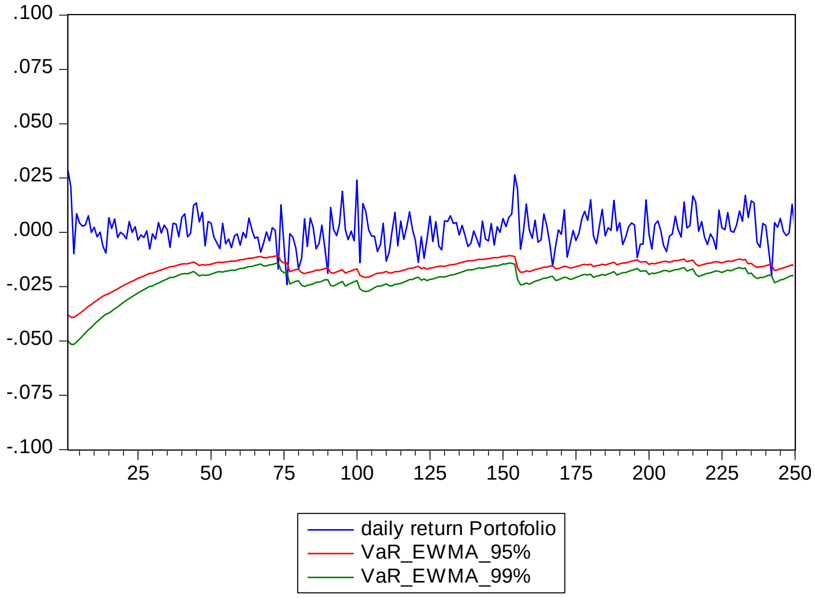


***Figure 6.*** *VaR estimates obtained from EWMA Model*

Source: author calculations

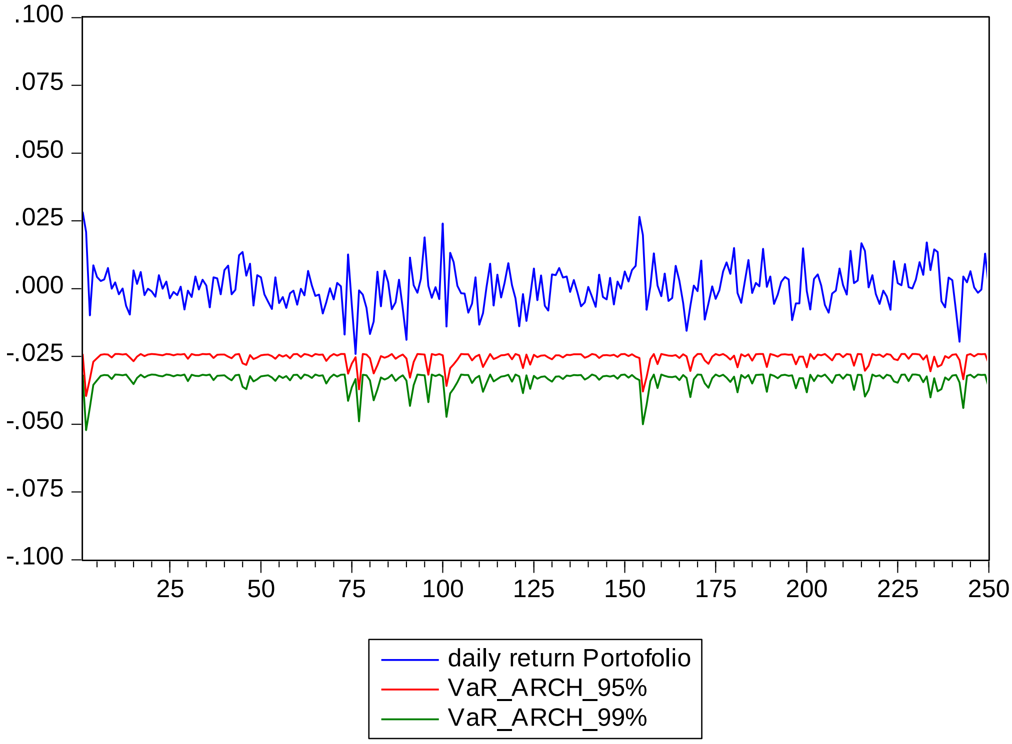


***Figure 7.*** *VaR estimates obtained from ARCH Model*
Source: author calculations

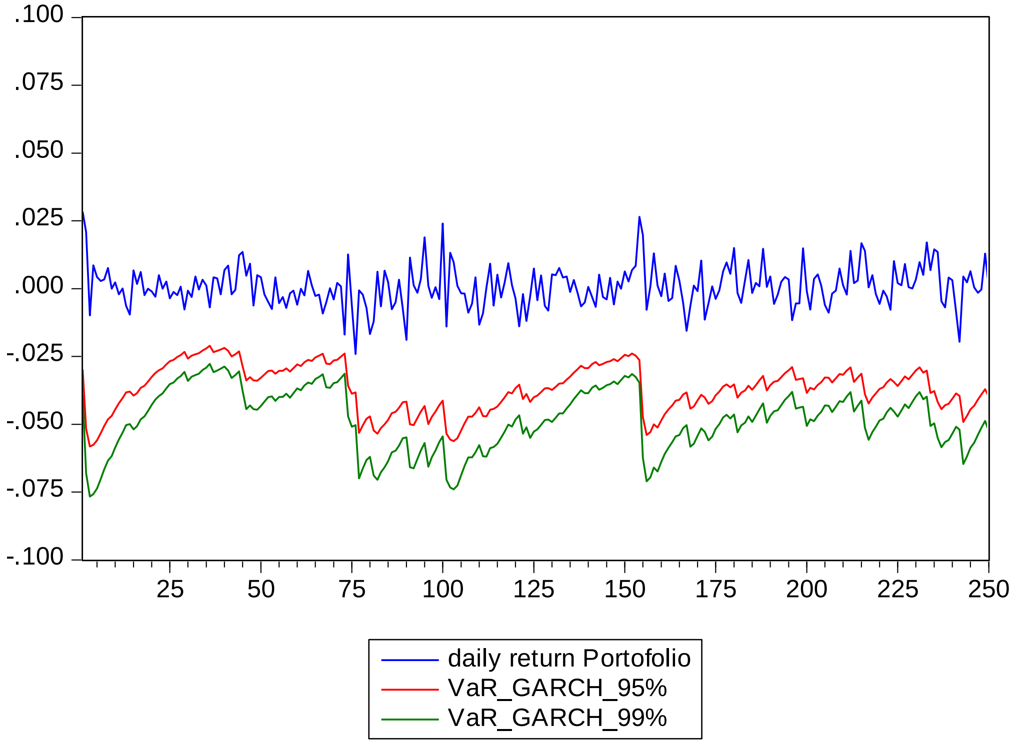


***Figure 8.*** *VaR estimates obtained from GARCH Model*
Source: author calculations

Given the above results, I conclude that the ARCH and GARCH models are more appropriate for estimating VaR than EWMA model. Also, from the above graphs, it can be observed that the GARCH model implies a lower cost of risk and for this reason this model is the most appropriate volatility forecasting model to estimate the Value-at-Risk.

## 5. Conclusions

This study was conducted to analyse the market risk (estimated by Value-at-Risk) on the Romanian capital market using modern econometric tools to estimate volatility, such as EWMA, GARCH models. I've worked with a period of 4 years, considering three representative indices of Romanian capital market. Heteroskedasticity models have proved extremely useful in modeling volatility. After repeated attempts, the best model was found to be GARCH model (1.1). Analyzing the results obtained through GARCH equation, I can draw the following conclusions:

- Coefficient of volatility is positive, indicating that when volatility increases, portfolio returns tend to increase;

- Coefficient that estimates ARCH effects in the data series analyzed, recorded a statistically significant amount. In other words, on the Romanian capital market, the periods characterized of high volatility continues throughout with high volatility, and vice versa.

- Coefficient which measures the asymmetry of the data series recorded a positive value, which suggests that negative shocks (bad news) generated less volatility than positive shocks (good news) on the Romanian capital market.

VaR depends on the volatility, time horizon and confidence interval for the continuous returns under analysis. Volatility tends to happen in clusters. The assumption that volatility remains constant at all times can be fatal. It is determined that the most recent data have asserted more influence on future volatility than past data. To emphasize this fact, recently, EWMA and GARCH models have become critical tools in financial applications.

Applying the test of „violation ratio" I've found that Value-at-Risk estimated by GARCH model was the most appropriate to estimate the risk of a portfolio of the 3 indices on the Romanian capital market. So, GARCH provides more accurate analysis than EWMA.This approach is useful for traders and risk managers to be able to forecast the future volatility on a certain market.

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